



pile and nodular diaphragm wall [12]. As a result of load tests, the nodular cast-in-place concrete pile and nodular diaphragm wall have large tension and compression resistance. The tension resistance at the nodular part and Under-reamed part shows a large value. In a review of last researches, it seems that they are all about a constant arrangement or specific type of groups, and still no comprehensive research is done on group arrangement or spacing of Under-reamed piles. In this research 4 types of pile group arrangements are considered for groups containing 3, 5 and 7 piles and compared together to find out an optimized pattern to reach the maximum bearing capacity in saturated sandy soil. Figure 1 shows these arrangements.

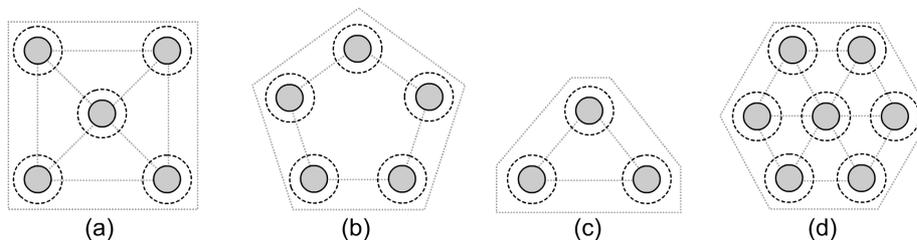


Figure 1. Pile group arrangements; (a) Square, (b) Pentagon, (c) Triangle, (d) Hexagon

## 2. MODELING VALIDATION

To validate the models, an illustrated example which was manually calculated in a handbook of Under-reamed pile foundations prepared by Devendra Sharma, M. P. Jain, Chandra Prakash (1978) [1], is first compared to a finite element model containing the same conditions of the example. Results were shown the same results in the both of numerical model and the example of Under-reamed pile. Equation 1 shows the formula which is used in this example [1], and diagram 1 shows a comparison between the results.

$$Q_u = A_p \left( \frac{1}{2} \cdot D \cdot \gamma \cdot N_\gamma + \gamma \cdot d_f \cdot N_q \right) + A_a \left[ \frac{1}{2} \cdot D_u \cdot n \cdot \gamma \cdot N_\gamma + \gamma \cdot N_q \cdot \sum_{r=1}^{r=n} dr \right] + \frac{1}{2} \cdot \pi \cdot D \cdot \gamma \cdot K \cdot \tan \delta \times (d_1^2 + d_f^2 + d_n^2) \quad (1)$$

Where  $Q_u$  (kg) is ultimate bearing capacity,  $A_p$  (cm<sup>2</sup>) is cross sectional area of pile stem at toe level and it's equal to:  $(\pi/4) \cdot D^2$ ,  $D$  (cm) is stem diameter,  $A_a$  (cm<sup>2</sup>) is cross sectional area of pile at bulb level and it's equal to:  $(\pi/4) \cdot (D_u^2 - D^2)$ ,  $D_u$  (cm) is Under-reamed bulb diameter,  $N_\gamma$ ,  $N_q$  are bearing capacity factors depending upon the angle of internal friction  $\phi$ , given in existing tables in foundation engineering books [1], [9],  $n$  is number of Under-reamed bulbs,  $d_r$  (cm) is depth of the center of different Under-reamed bulbs below ground level,  $d_f$  (cm) is depth of below ground level,  $K$  is earth pressure coefficient (usually taken 1.75 for sandy soils),  $\delta$  is the angle of wall friction (may be taken equal to the angle of internal friction  $\phi$ ,  $d_1$  (cm) is depth of the center of first Under-reamed bulb,  $d_n$  (cm) is depth of the center of the last Under-reamed bulb. For working out the uplift loads from the formula, there will be no contribution by the pile toe as the direction of load application is reversed. Thus the first term of the formula will not occurs. However the ultimate loads by this method may be found slightly on higher side. For determining the safe loads in uplift the recommended factor of safety is 3. and if the pile is a single bulb pile, the third term is neglected [1]. Figure 2 shows a comparison between the model and the example of an Under-reamed pile under tensile loading.

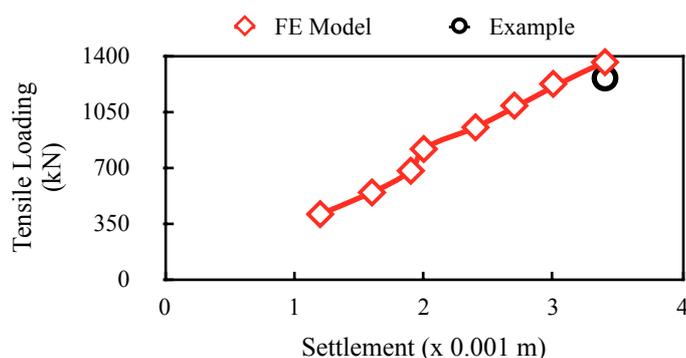


Figure 2. Comparison between the model and the example of an Under-reamed pile.

### 3. NUMERICAL MODELING METHODOLOGY

Modeling process was done using Plaxis 3D Foundation, it's a Finite Element based software. As shown in figure 3, there is some considerations for piles and pile caps specifications, the modeling process in this research was done under these standards [1], [2], [3], [7]. In the models applied in this research all piles' diameters were 0.5 meter, piles maximum depth were 6.2 meter, single Under-reamed bulbs were located at the end of the pile stem and the bulbs' maximum diameter were 1.4 meter, the bucket length were 0.7 meter. Thickness of the caps were 1 meter and they are located on the floor not in the soil. Plaxis 3D foundation has no option to model an Under-reamed pile, so it should be considered in different layers.

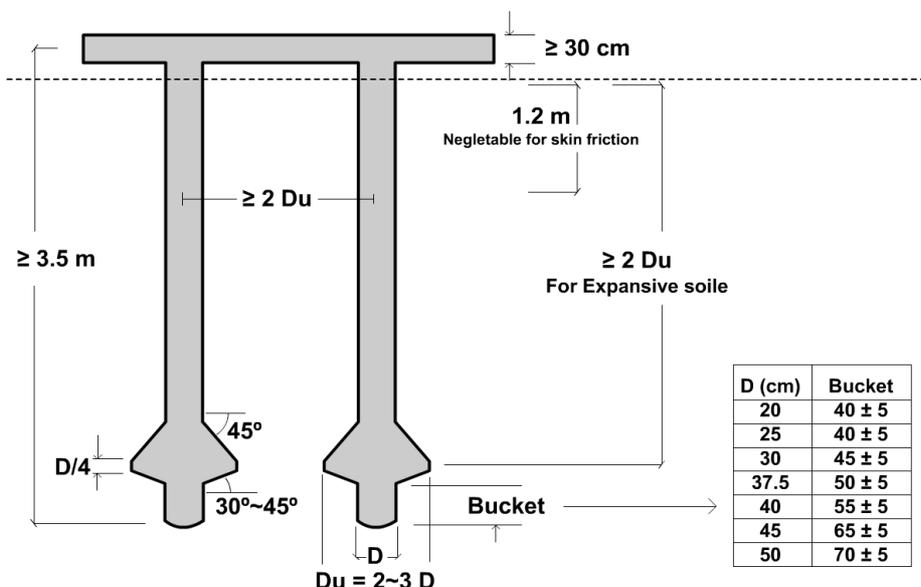


Figure 3. Schematic section of considerations for Under-reamed piles is group [1].

Modeling process was started by making one pile models and then groups containing uniform stem piles and Under-reamed piles. Modeling was continued by making 4 and 5 piles groups respectively with different arrangements and spacing from spacing/diameter (S/D) ratios of 6 to 12. 5 piles group was arranged in square shape with a central pile. Modeling general conditions were the same for all of the models, and outputs variables such as ultimate bearing capacities, settlements, etc. recorded step-by-step in a spreadsheet for next analysis and comparing. Results for square arrangement is discussed elaborately in the next section. Soil and pile specifications, material modes, stiffness characteristics and interfaces are shown in table 1. Material mode s is Linear elastic and Mohr-Coulomb for piles and soil respectively.

Table 1- Specifications of materials which is considered in the FE models [Plaxis 3D Foundation]

		Pile	Soil
Material Mode		Linear elastic	Mohr-Coulomb
General Properties	$\gamma_{\text{unsat}}$	24	17
	$\gamma_{\text{sat}}$	24	20
Stiffness	E (kN/m <sup>2</sup> )	2.920 * 10 <sup>7</sup>	1.5 * 10 <sup>4</sup>
	$\nu$ (nu)	0.300	0.300
	C	-	1

		Pile	Soil
Strength	$\phi$ (Phi)	-	32°
	$\psi$ (psi)	-	2°
Interface	Rigidity ( $R_{inter}$ )	(Rigid) 1	0.7

Finite edges of models formed by 20x20 area in plan and 15m of depth. Caps were allowed to have about 10m space to the edges. There are roller fixities on the edges. Figure 4 shows a 3D mesh generated by Plaxis 3D Foundation, it's necessary to generate 3D mesh before calculation phase. Figure 5 shows an output example for a square arrangement model. As the figure shows, there are more stress at the opposite direction of loading in the Under-reamed bulbs area. The extreme value for this loading phase is  $1.85 \times 10^3 \text{ kN/m}^2$ .

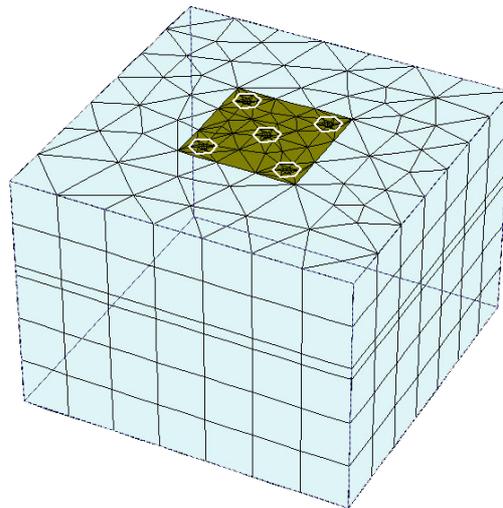


Figure 4. 3D Mesh for 5 pile square arrangement, generated by Plaxis 3D Foundation software

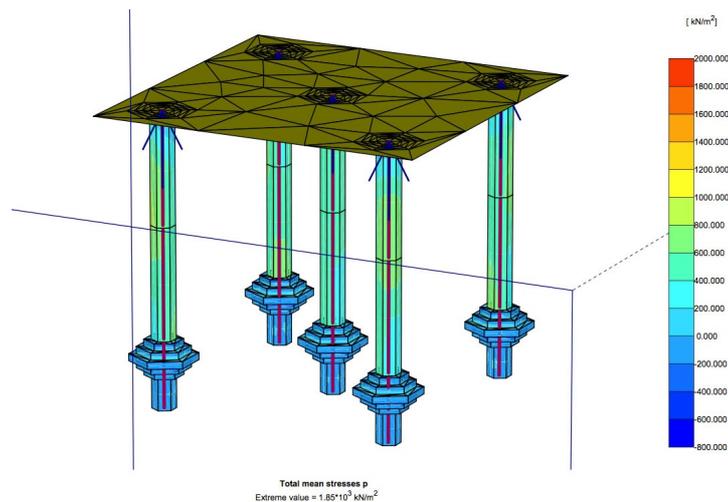


Figure 5. Output program for stresses values of 5 Under-reamed piles group.

Then next set of models with pentagon plan geometry was arranged to complete the modeling, rest of the values were recorded precisely. Also a set of triangle models with 3 piles were made and a set of hexagon

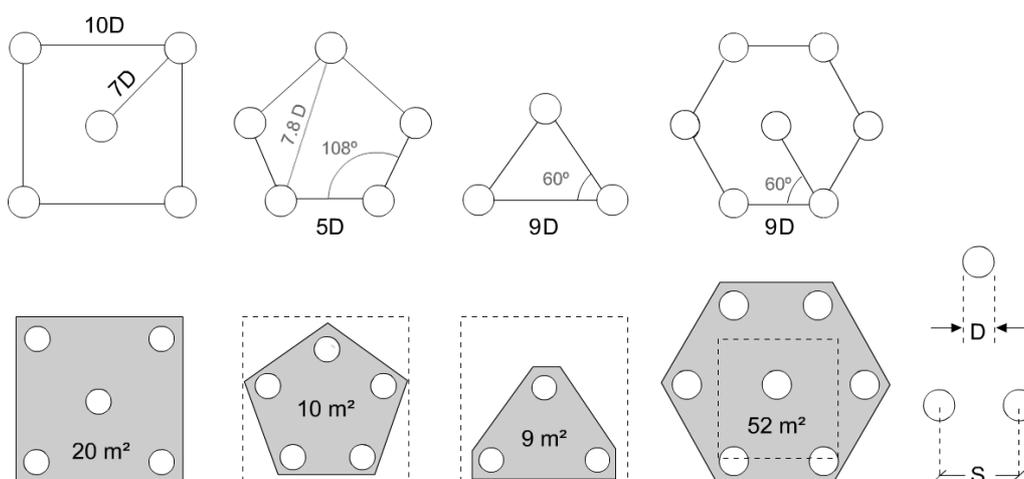
models for 6 and 7 piles were made as the finishing stage of modeling. Results for pentagon, triangle and hexagon arrangements is discussed elaborately in the next sections.

#### 4. PARAMETRIC ANALYSIS

Quantitative parameters of the models were compared together to derive a relation between these parameters in order to find out the best arrangement pattern. For all the arrangements, increasing the space between piles, their bearing capacities were increased gradually. After a particular spacing the groups were loosed the bearing capacity. Table 2 shows S/D ranges for some types of arrangements modeled in this research, also area ranges is shown for the groups. Areas is compared together in order to efficiency of every arrangement. Figure 6 shows a comparison of geometry between Square, Pentagon, Triangle and Hexagon arrangement, angle variations are shown in this figure for every arrangement.

**Table 2- Range of Spacing/Diameter (S/D) ratio in model series for 3, 5 and 7 piles groups**

Arrangement	Number of piles	Range of S/D ratio	Area range (m <sup>2</sup> )
Square	5	6 - 16	9 - 64
Pentagon	5	3 - 10	3.9 - 43
Triangle	3	3 - 10	0.97 - 10.8
Hexagon	7	8 - 12	41.5 - 93



**Figure 6. Comparison of geometry between Square, Pentagon, Triangle and Hexagon arrangements.**

#### 5. RESULTS AND DISCUSSIONS

Outputs were shown that piles which were located at the optimum arrangement were working together uniformly well. So each pile could withstands more loads in optimum arrangement, as shown in the figure 7 and 8, that's why every single pile in pentagon could be subjected to about 23 tons and in the square arrangement every pile could withstand just about 15 tons. Pentagonal arrangement in a comparison with square arrangement of 5 piles, could be subjected to bigger loads up to 16%, also in a same loading condition in the comparison it could save 40% of materials and 46% of area. Another comparison was applied for triangular arrangement containing 3 piles and square arrangement containing 4 piles, that was shown the triangular arrangement more efficient in bearing capacity at equal area (23%) and more efficient in area at equal loading condition (28%). Hexagonal arrangement was not successful arrangement in relation to area efficiency, it was compared to a simple square arrangement of 6 piles and with a symmetric hexagonal

arrangement of 6 piles, it could be subjected to just  $\frac{1}{4}$  of square arrangement loading in unit area and  $\frac{1}{3}$  of loading quantity of symmetric hexagonal arrangement. Pentagonal arrangement was also compared to the symmetric hexagonal arrangement of 6 piles, pentagon was appeared still better than hexagon and the amount of loading per unit area was increased to 90%. Figure 7 shows a shading view of vertical displacement in output program for two Under-reamed piles next together in pentagon arrangement at spacing/diameter (S/D) ratio of 5. The extreme value of this model is 5.48mm which is shown in red color. Also the failure zone could be visible by shear strain shading output in Plaxis 3D Foundation.

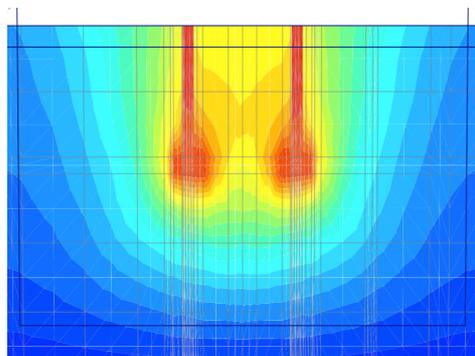


Figure 7. Shading view for vertical displacements of loading phase for pentagon arrangement.

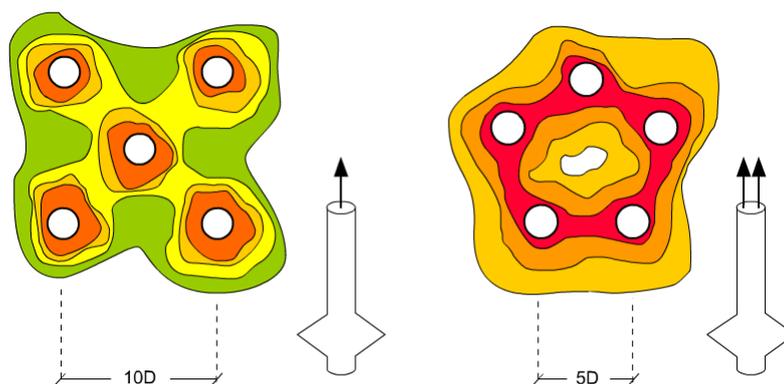


Figure 8. Comparison of square and pentagon bearing capacities.

As discussed in section 4, modeling process applied according to parameters of table 2, the optimum S/D ratios for the arrangements were resulted as shown in table 3. Pentagon have the smallest number of S/D ratio between them. Figures 9 shows tensile bearing capacities corresponding to the S/D ratio, and figure 10 shows area efficiency the arrangements. As figure 9 shows, pentagonal arrangement is reached the maximum capacity in S/D ratio of 5. The diagram shows that pentagon is reached the optimum arrangement in lowest S/D ratio in whole curves. Triangular, Hexagonal and square arrangements are reached to the optimum point at S/D ratios about 9 to 10.

Table 3- Optimum Spacing/Diameter (S/D) ratio for 3, 5 and 7 piles groups

Arrangement	Number of piles	Optimum S/D ratio
Square	5	10
Pentagon	5	5
Triangle	3	9
Hexagon	7	9

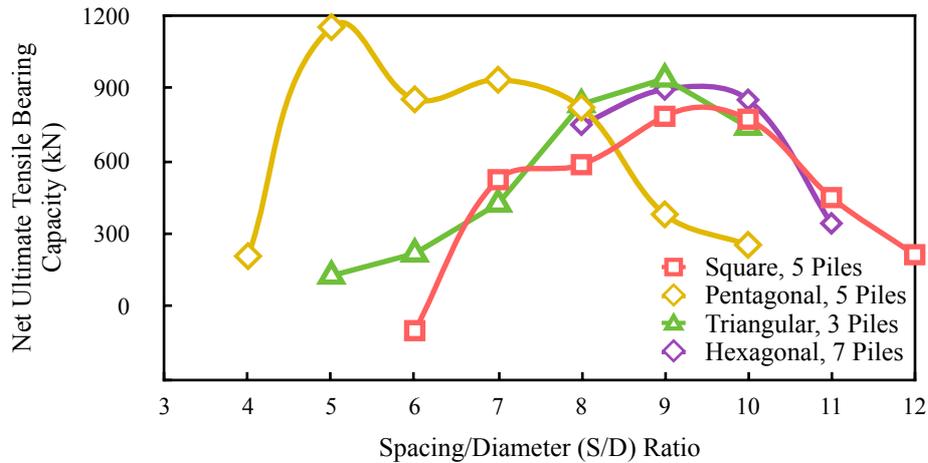


Figure 9. Ultimate tensile bearing capacity for different S/D ratios of Under-reamed pile groups.

Their area efficiencies are compared in figure 10. Pentagonal arrangement had more tensile bearing capacities in smaller areas, for example to reach 800 kN, pentagon needs just 15 square meter, while in this area the square group could withstand just about 500 kN. In this diagram also triangular arrangement is shown as an efficient arrangement in small areas.

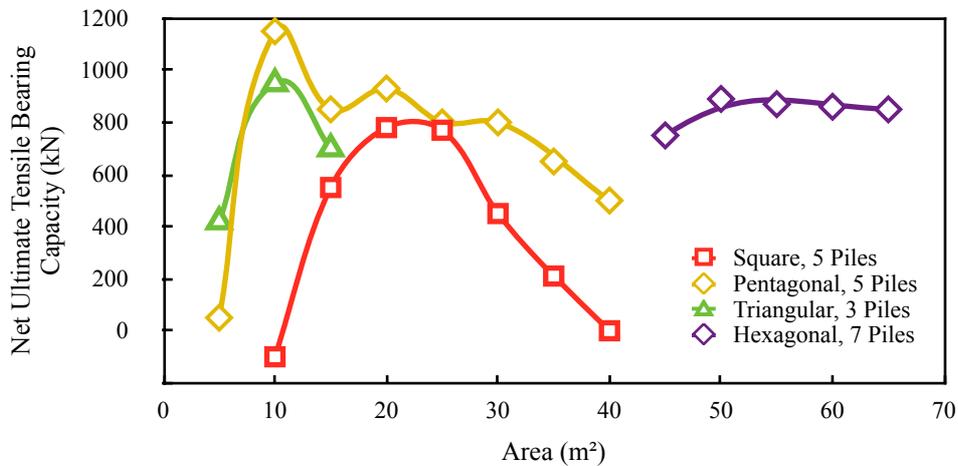


Figure 10. Area efficiencies for different arrangements of Under-reamed pile groups in sandy soil.

In a comparison between pentagon and square with same number of piles with different arrangement, pentagon shows smaller number of S/D ratio, and smaller amounts of areas for withstand larger amounts of bearing capacities. Comparison between these models couldn't be generalized for bigger groups of them. In a quick review of figure 10, it's obvious that hexagonal arrangement for 7 piles is not efficient at all comparing to even smaller number of piles in pentagonal arrangement.

## 6. CONCLUSIONS

In this research some series of models containing Under-reamed pile groups with different arrangements and spacing between piles which were subjected to tensile loading, were applied using Finite Element method and were compared together. Analyses contain some results as below:



1. Optimum S/D ratios were obtained 10, 5, 9 and 9 respectively for square arrangement of Under-reamed pile group (containing 5 piles), pentagon arrangement (containing 5 piles), triangle arrangement (containing 3 piles) and hexagon arrangement (containing 7 piles). This number was related to angles between piles in plan.
2. Ultimate bearing capacity for pentagonal arrangement was increased more than 16 percent comparing to the square arrangement in the same area. Also Pentagon could be applied in much lower concrete consumption than square arrangement (more than 40%). So pentagon arrangement appeared more efficient than square arrangement for a group with 5 Under-reamed piles in saturated sandy soil.
3. Triangular arrangement could be subjected to higher loading quantities up to 23% comparing to symmetric square arrangement with 4 piles. To achieve a certain amount of bearing capacity comparing to square arrangement, area was decreased 28% in triangular arrangement.
4. It is able to withstand greater loads in a smaller area, it cause less operation volume in project process.
5. Asymmetry may sometimes improve tensile bearing capacity potentials of an Under-reamed pile group.
6. In the examined models in this research, pentagonal and triangular arrangements had a very good results for tensile bearing capacity per area unit or concrete usage compared to other types of arrangements with similar number of piles.

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